

Practice Problem Set 1

Question 1 “A risk-neutral individual with the VN & M utility representation makes investment decisions in accordance with the principle of maximum expected return, rather than with that of maximum expected utility.” Is this statement true? Explain your answer by means of a graph.

Question 2 An individual with the VN & M utility representation and Bernoulli utility function $u(x) = (x/10)^2$ is known to be indifferent between two lotteries A and B, which offer the following prizes:

Lottery A		Lottery B	
Prob.	Prize	Prob.	Prize
0.5	110	p	90
0.5	130	1-p	150

Find the value of p.

Question 3 Assume an investor with the following utility function:

Return (\$)	Utility
-2,000	-600
-1,000	-150
0	0
1,000	80
2,000	150
3,000	210
4,000	250
5,000	280
10,000	340

(a) Graph the investor's utility function.

(b) Will this investor prefer a sure payment of \$2,000 or to invest in a project with an equal chance to be a complete loss or to generate a cash flow with a PV of \$5,000?

(c) What is the maximum amount the investor would agree to invest in a project with a cash flow with an equal probability of a PV of \$1,000 or a PV of \$ 10,000?

(d) Answer parts (b) and (c) graphically.

Question 4 The forecast of the rate of return and the assessment of the possible states of the world for two stocks are given in the following table:

State of the Economy	Probability	Stock A: Return	Stock B: Return
Moderate Recession	.05	-.02	-.20
Slight Recession	.15	-.01	-.10
2% Growth	.60	.15	.15
3% Growth	.20	.15	.30

(a) What are the Expected Returns on Stock A and Stock B? Show that the standard deviations of the two stocks are $\sigma_A = 0.065$ and $\sigma_B = 0.1392$ respectively.

(b) Suppose in this economy investors can borrow and lend at the risk-less interest rate $r_f = 0.05$. Also, suppose that the only feasible portfolios include only one of the risky assets.

(i) In the (Expected Return, Standard Deviation) plane, draw the Risk-Return tradeoff line for all portfolios that include $x\%$ of the risk-less asset and $(1-x)\%$ of Stock A. What is the equation of the Risk-Return tradeoff line? What is the Standard Deviation of a portfolio that has an expected return of 7%? What is the Standard Deviation of a portfolio that has an expected return of 15%?

(ii) In the (Expected Return, Standard Deviation) plane, draw the Risk-Return tradeoff line for all portfolios that include $x\%$ of the risk-less asset and $(1-x)\%$ of Stock B. What is the equation of the Risk-Return tradeoff line? What is the Standard Deviation of a portfolio that has an expected return of 7%? What is the Standard Deviation of a portfolio that has an expected return of 15%?

(iii) Suppose you are a Mean-Variance investor. Which type of portfolios between those you found in part (i) and (ii) would you choose? What further information would you need to determine your Optimal portfolio?

Question 5 Distinguish between an efficient frontier and an optimal portfolio.

Question 6 Assume you can either invest all of your resources in one of two securities A or B, or, alternatively, you can diversify your investment between the two. The distribution of the returns are as follows:

Probability	Stock A: Return	Stock B: Return
.5	-10%	-20%
.5	50%	60%

Assume that the correlation between the two returns is zero.

- (a) Compute each security's expected return, variance and standard deviation.
- (b) Calculate the probability distribution of the returns on a mixed portfolio comprising equal proportions of securities A and B. Also, calculate the expected return, variance and standard deviation.
- (c) Calculate the expected return and the variance of a mixed portfolio comprising 75% of security A and 25% of security B.